Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

1. Determining a Solution Determine whether the function $y = x^3$ is a solution of the differential equation $2xy' + 4y = 10x^3$.

2. Determining a Solution Determine whether the function $y = 2 \sin 2x$ is a solution of the differential equation $y''' - 8y = 0$.

Finding a General Solution **In Exercises 3–8, use integration to find a general solution of the differential equation.**

3. $\dfrac{dy}{dx} = 4x^2 + 7$

4. $\dfrac{dy}{dx} = 3x^3 - 8x$

5. $\dfrac{dy}{dx} = \cos 2x$

6. $\dfrac{dy}{dx} = 2 \sin x$

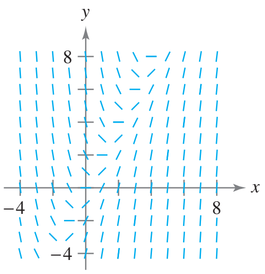
7. $\dfrac{dy}{dx} = e^{2-x}$

8. $\dfrac{dy}{dx} = 2e^{3x}$

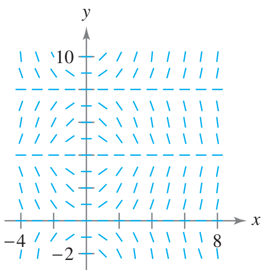
Slope Field **In Exercises 9 and 10, a differential equation and its slope field are given. Complete the table by determining the slopes (if possible) in the slope field at the given points.**

x	-4	-2	0	2	4	8
y	2	0	4	4	6	8
dy/dx						

9. $\dfrac{dy}{dx} = 2x - y$



10. $\dfrac{dy}{dx} = x \sin\left(\dfrac{\pi y}{4}\right)$



Slope Field **In Exercises 11 and 12, (a) sketch the slope field for the differential equation, and (b) use the slope field to sketch the solution that passes through the given point. Use a graphing utility to verify your results. To print a blank graph, go to *MathGraphs.com*.**

11. $y' = 2x^2 - x, \quad (0, 2)$

12. $y' = y + 4x, \quad (-1, 1)$

Euler's Method **In Exercises 13 and 14, use Euler's Method to make a table of values for the approximate solution of the differential equation with the specified initial value. Use n steps of size h.**

13. $y' = x - y, \quad y(0) = 4, \quad n = 10, \quad h = 0.05$

14. $y' = 5x - 2y, \quad y(0) = 2, \quad n = 10, \quad h = 0.1$

Solving a Differential Equation **In Exercises 15–20, solve the differential equation.**

15. $\dfrac{dy}{dx} = 2x - 5x^2$

16. $\dfrac{dy}{dx} = y + 8$

17. $\dfrac{dy}{dx} = (3 + y)^2$

18. $\dfrac{dy}{dx} = 10\sqrt{y}$

19. $(2 + x)y' - xy = 0$

20. $xy' - (x + 1)y = 0$

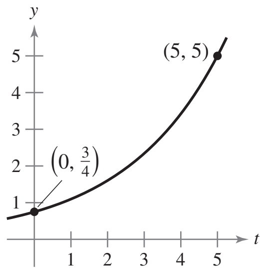
Writing and Solving a Differential Equation **In Exercises 21 and 22, write and solve the differential equation that models the verbal statement.**

21. The rate of change of y with respect to t is inversely proportional to the cube of t.

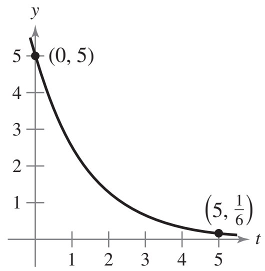
22. The rate of change of y with respect to t is proportional to $50 - t$.

Finding an Exponential Function **In Exercises 23–26, find the exponential function $y = Ce^{kt}$ that passes through the two points.**

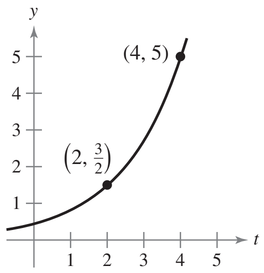
23.



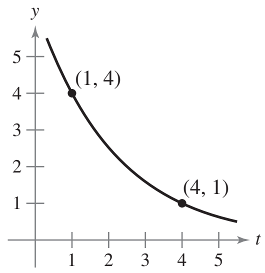
24.



25.



26.



27. Air Pressure Under ideal conditions, air pressure decreases continuously with the height above sea level at a rate proportional to the pressure at that height. The barometer reads 30 inches at sea level and 15 inches at 18,000 feet. Find the barometric pressure at 35,000 feet.

28. Radioactive Decay Radioactive radium has a half-life of approximately 1599 years. The initial quantity is 15 grams. How much remains after 750 years?

29. Population Growth A population grows continuously at the rate of 1.85%. How long will it take the population to double?

Copyright 2012 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s). Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.

30. Compound Interest Find the balance in an account when \$1000 is deposited for 8 years at an interest rate of 4% compounded continuously.

31. Sales The sales S (in thousands of units) of a new product after it has been on the market for t years is given by

$S = Ce^{k/t}$.

(a) Find S as a function of t when 5000 units have been sold after 1 year and the saturation point for the market is 30,000 units (that is, $\lim_{t\to\infty} S = 30$).

(b) How many units will have been sold after 5 years?

32. Sales The sales S (in thousands of units) of a new product after it has been on the market for t years is given by

$S = 25(1 - e^{kt})$.

(a) Find S as a function of t when 4000 units have been sold after 1 year.

(b) How many units will saturate this market?

(c) How many units will have been sold after 5 years?

Finding a General Solution Using Separation of Variables **In Exercises 33–36, find the general solution of the differential equation.**

33. $\dfrac{dy}{dx} = \dfrac{5x}{y}$ **34.** $\dfrac{dy}{dx} = \dfrac{x^3}{2y^2}$

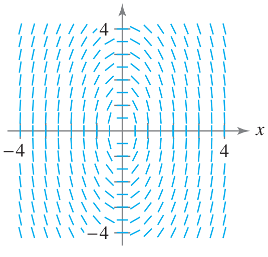
35. $y' - 16xy = 0$ **36.** $y' - e^y \sin x = 0$

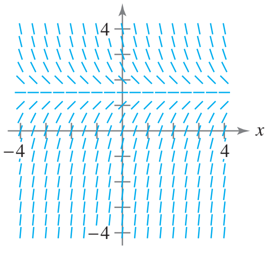
Finding a Particular Solution Using Separation of Variables **In Exercises 37–40, find the particular solution that satisfies the initial condition.**

	Differential Equation	Initial Condition
37.	$y^3y' - 3x = 0$	$y(2) = 2$
38.	$yy' - 5e^{2x} = 0$	$y(0) = -3$
39.	$y^3(x^4 + 1)y' - x^3(y^4 + 1) = 0$	$y(0) = 1$
40.	$yy' - x\cos x^2 = 0$	$y(0) = -2$

Slope Field **In Exercises 41 and 42, sketch a few solutions of the differential equation on the slope field and then find the general solution analytically. To print an enlarged copy of the graph, go to *MathGraphs.com*.**

41. $\dfrac{dy}{dx} = -\dfrac{4x}{y}$ **42.** $\dfrac{dy}{dx} = 3 - 2y$





Using a Logistic Equation **In Exercises 43 and 44, the logistic equation models the growth of a population. Use the equation to (a) find the value of k, (b) find the carrying capacity, (c) find the initial population, (d) determine when the population will reach 50% of its carrying capacity, and (e) write a logistic differential equation that has the solution $P(t)$.**

43. $P(t) = \dfrac{5250}{1 + 34e^{-0.55t}}$

44. $P(t) = \dfrac{4800}{1 + 14e^{-0.15t}}$

Solving a Logistic Differential Equation **In Exercises 45 and 46, find the logistic equation that passes through the given point.**

45. $\dfrac{dy}{dt} = y\left(1 - \dfrac{y}{80}\right)$, $(0, 8)$

46. $\dfrac{dy}{dt} = 1.76y\left(1 - \dfrac{y}{8}\right)$, $(0, 3)$

47. Environment A conservation department releases 1200 brook trout into a lake. It is estimated that the carrying capacity of the lake for the species is 20,400. After the first year, there are 2000 brook trout in the lake.

(a) Write a logistic equation that models the number of brook trout in the lake.

(b) Find the number of brook trout in the lake after 8 years.

(c) When will the number of brook trout reach 10,000?

48. Environment Write a logistic differential equation that models the growth rate of the brook trout population in Exercise 47. Then repeat part (b) using Euler's Method with a step size of $h = 1$. Compare the approximation with the exact answer.

Solving a First-Order Linear Differential Equation **In Exercises 49–54, solve the first-order linear differential equation.**

49. $y' - y = 10$

50. $e^x y' + 4e^x y = 1$

51. $4y' = e^{x/4} + y$

52. $\dfrac{dy}{dx} - \dfrac{5y}{x^2} = \dfrac{1}{x^2}$

53. $(x - 2)y' + y = 1$

54. $(x + 3)y' + 2y = 2(x + 3)^2$

Finding a Particular Solution **In Exercises 55 and 56, find the particular solution of the differential equation that satisfies the initial condition.**

	Differential Equation	Initial Condition
55.	$y' + 5y = e^{5x}$	$y(0) = 3$
56.	$y' - \left(\dfrac{3}{x}\right)y = 2x^3$	$y(1) = 1$

Copyright 2012 Cengage Learning. All Rights Reserved. May not be copied, scanned, or duplicated, in whole or in part. Due to electronic rights, some third party content may be suppressed from the eBook and/or eChapter(s). Editorial review has deemed that any suppressed content does not materially affect the overall learning experience. Cengage Learning reserves the right to remove additional content at any time if subsequent rights restrictions require it.